Engineering Example A2:
Drill Requirement Validation

This worksheet focuses on the diameter of the drive shaft, the speed of the drive shaft, and the output horsepower of a gasoline engine to meet specific drill torque and force requirements.

Field: Mechanical Engineering

Features used:
User-defined functions
Arrays
Vector and matrix functions
Vectorize
Advanced programming
Multiple inputs and outputs
Design of Experiments
Excel component

This example is based on an example provided by John Sheehan of PTC.

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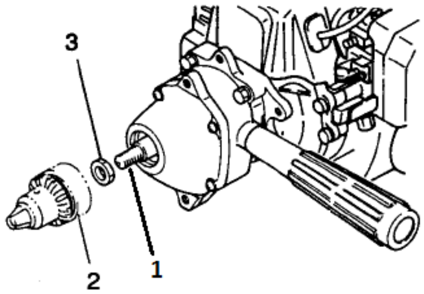
REQUIREMENTS
Engine: Two stroke, single cylinder,
Drive Shaft Speed Range: 300 rpm to 400 rpm
Horse Power Range 1.1 hp to 1.9 hp
Drive Shaft Diameter Range: 22mm to 28mm
Minimum Torque: 30 N*m
Minimum Force: 2000 N
The gears must spin the drive shaft at a speed of between 300 rpm and 400 rpm. Ensure that the engine horse power and drive shaft diameter are sufficient to generate the minimum torque and minimum force on the drill bit.



1. Drive Shaft
2. Chuck
3. Collar

The PTC Mathcad default definition of ***rpm*** is 360 deg/min. Change it to 1 revolution/min. Use the Unit label.

$rpm = 360.00 \frac{deg}{min}$ $rpm := \frac{1}{min}$

$\mathrm{Drive\_Shaft}_{\mathrm{speed}} := 350\ rpm$ $\mathrm{Engine}_{\mathrm{hp}} := 1.5\ hp$

$\mathrm{Drive\_Shaft}_{\mathrm{Diameter}} := 24\ mm$ $\mathrm{MinTorque} := 30\ N \cdot m$ $\mathrm{MinForce} := 2000\ N$

Create functions to test whether or not the torque and force meet minimum requirements

$\mathrm{Test1}(\mathrm{Torque}) := \mathrm{if}(\mathrm{Torque} > \mathrm{MinTorque}, \text{“Meets”}, \text{“Doesn't Meet”})$

$\mathrm{Test2}(\mathrm{Force}) := \mathrm{if}(\mathrm{Force} > \mathrm{MinForce}, \text{“Meets”}, \text{“Doesn't Meet”})$

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Formulas

$$\mathbf{Torque} = \frac{hp}{2 \cdot \pi \cdot rpm} \qquad \mathbf{Force} = \frac{\mathbf{Torque}}{\mathbf{Radius}}$$

Check Torque

$$\text{Torque} := \frac{\text{Engine}_{\text{hp}}}{\text{Drive\_Shaft}_{\text{speed}} \cdot 2\,\pi} = 30.52\ N \cdot m$$

$$\text{Test1}(\text{Torque}) = \text{“Meets”}$$

Check Force

$$\text{Force} := \frac{\text{Torque}}{\dfrac{\text{Drive\_Shaft}_{\text{Diameter}}}{2}} = 2543.182\ N$$

$$\text{Test2}(\text{Force}) = \text{“Meets”}$$

The current values of 1.5 hp, 350 rpm, and 24mm meet the minimum torque and force requirements.

TRADEOFF ANALYSIS:

We are trying to discover which settings for the 3 variables (speed of a drive shaft, horsepower applied to the drive shaft, and diameter of the drive shaft) will create the greatest force on a bit applied by the shaft. In this example each of the 3 variables have 5 possible settings. Test all possible combinations.

Use a table for the input. Remember that the top row of a table is for the variable name, and the second row is for the units. The data begins on the third row.

Speed	HorsePower	Diameter
(rpm)	(hp)	(mm)
300	1.1	20
325	1.3	22
350	1.5	24
375	1.7	26
400	1.9	28

$$\text{Speed} = \begin{bmatrix} 300.00 \\ 325.00 \\ 350.00 \\ 375.00 \\ 400.00 \end{bmatrix} rpm$$

$$\text{HorsePower} = \begin{bmatrix} 1.10 \\ 1.30 \\ 1.50 \\ 1.70 \\ 1.90 \end{bmatrix} hp$$

$$\text{Diameter} = \begin{bmatrix} 20.00 \\ 22.00 \\ 24.00 \\ 26.00 \\ 28.00 \end{bmatrix} mm$$

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In order to check all of the possible combinations, use the ***fullfact*** function. This is a Design of Experiments full factorial matrix generator. We will use ***fullfact*** for 3 variables with 5 levels. The last 3 columns are of interest. We will use the last three columns to tell us which value for Speed, HP and Diameter to test in each calculation. There are three variables each with five values so the full factorial totals 125 possibilities.

$ff := fullfact(3,5)$ $\quad ORIGIN = 1.00$ $\quad rows(ff) = 126.00$

$$ff = \begin{bmatrix} \text{“Run”} & \text{“Block”} & \text{“A”} & \text{“B”} & \text{“C”} \\ 1 & 1 & 0 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 \\ 3 & 1 & 2 & 0 & 0 \\ 4 & 1 & 3 & 0 & 0 \\ 5 & 1 & 4 & 0 & 0 \\ 6 & 1 & 0 & 1 & 0 \\ 7 & 1 & 1 & 1 & 0 \\ 8 & 1 & 2 & 1 & 0 \\ 9 & 1 & 3 & 1 & 0 \\ 10 & 1 & 4 & 1 & 0 \\ 11 & 1 & 0 & 2 & 0 \\ & & & & \vdots \end{bmatrix} \qquad ff = \begin{bmatrix} \vdots & & & & \\ 114.00 & 1.00 & 3.00 & 2.00 & 4.00 \\ 115.00 & 1.00 & 4.00 & 2.00 & 4.00 \\ 116.00 & 1.00 & 0.00 & 3.00 & 4.00 \\ 117.00 & 1.00 & 1.00 & 3.00 & 4.00 \\ 118.00 & 1.00 & 2.00 & 3.00 & 4.00 \\ 119.00 & 1.00 & 3.00 & 3.00 & 4.00 \\ 120.00 & 1.00 & 4.00 & 3.00 & 4.00 \\ 121.00 & 1.00 & 0.00 & 4.00 & 4.00 \\ 122.00 & 1.00 & 1.00 & 4.00 & 4.00 \\ 123.00 & 1.00 & 2.00 & 4.00 & 4.00 \\ 124.00 & 1.00 & 3.00 & 4.00 & 4.00 \\ 125.00 & 1.00 & 4.00 & 4.00 & 4.00 \end{bmatrix}$$

$Indices := submatrix(ff, 2, rows(ff), 3, 5) + ORIGIN$

The ***submatrix*** function above uses matrix "ff" and takes rows 2 to 126 (The first row is a header row), and columns 3 to 5. Remember that ORIGIN is set to 1, so we added the value of ORIGIN to the numbers.

$$Indices = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 3 & 1 & 1 \\ 4 & 1 & 1 \\ 5 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 2 & 1 \\ 3 & 2 & 1 \\ 4 & 2 & 1 \\ 5 & 2 & 1 \\ 1 & 3 & 1 \\ & & \vdots \end{bmatrix} \qquad Indices = \begin{bmatrix} \vdots & & \\ 5 & 3 & 5 \\ 1 & 4 & 5 \\ 2 & 4 & 5 \\ 3 & 4 & 5 \\ 4 & 4 & 5 \\ 5 & 4 & 5 \\ 1 & 5 & 5 \\ 2 & 5 & 5 \\ 3 & 5 & 5 \\ 4 & 5 & 5 \\ 5 & 5 & 5 \end{bmatrix}$$

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Make a function that will use the values from the matrix "Indices" as indices to create a matrix with all 125 possible combinations.

The following function assumes that "speed", "horse", "diam", and "indices" are all matrices.

The program creates the local variables "f_idx", "h_idx", "d_idx", and "data".

The matrix "indices" has the values 1, 2, 3, 4 and 5 in various combinations. These values will be used as indeces for the following local variables.

Variable f_idx takes values from column 1 of "indeces".
Variable h_idx takes values from column 2 of "indeces".
Variable d_ids takes values from column 3 of "indeces".
Each of these variables begins at row 1 and continues to the number of rows in "indices".

The local variable "data" is created using the previous three local variables as indeces to extract data from "force", "horse", and "diam". Column 1 uses the values from "force". Column 2 uses the values from "horse". and Column 3 uses the values from "diam".

```
Show_Data(speed, horse, diam, indices) :=
    for i ∈ ORIGIN .. rows(indices)
        f_idx ← indices[i, ORIGIN]
        h_idx ← indices[i, ORIGIN + 1]
        d_idx ← indices[i, ORIGIN + 2]
        data[i, ORIGIN] ← speed[f_idx]
        data[i, ORIGIN + 1] ← horse[h_idx]
        data[i, ORIGIN + 2] ← diam[d_idx]
    return data
```

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Use the Show_Data function to create a matrix with the input data.

$$\mathrm{Input\_Data} := \mathrm{Show\_Data}(\mathrm{Speed}, \mathrm{HorsePower}, \mathrm{Diameter}, \mathrm{Indices})$$

$$\mathrm{Input\_Data} = \begin{array}{r} 0 \\ 1 \\ \vdots \\ 124 \end{array} \begin{bmatrix} 5.00\ \dfrac{1}{s} & 820.27\ W & 0.02\ m \\ 5.42\ \dfrac{1}{s} & 820.27\ W & 0.02\ m \\ & \vdots & \end{bmatrix}$$

$$\mathrm{Input\_Data}^{\langle ORIGIN \rangle} = \begin{array}{r} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ \vdots \\ 124 \end{array} \begin{bmatrix} 300.00 \\ 325.00 \\ 350.00 \\ 375.00 \\ 400.00 \\ 300.00 \\ 325.00 \\ 350.00 \\ 375.00 \\ 400.00 \\ 300.00 \\ 325.00 \\ \vdots \end{bmatrix} rpm \quad \mathrm{Input\_Data}^{\langle ORIGIN+1 \rangle} = \begin{array}{r} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ \vdots \\ 124 \end{array} \begin{bmatrix} 1.10 \\ 1.10 \\ 1.10 \\ 1.10 \\ 1.10 \\ 1.30 \\ 1.30 \\ 1.30 \\ 1.30 \\ 1.30 \\ 1.50 \\ 1.50 \\ \vdots \end{bmatrix} hp$$

$$\mathrm{Input\_Data}^{\langle ORIGIN+2 \rangle} = \begin{array}{r} 0 \\ \vdots \\ 17 \\ 18 \\ 19 \\ 20 \\ 21 \\ 22 \\ 23 \\ 24 \\ 25 \\ 26 \\ 27 \\ \vdots \\ 124 \end{array} \begin{bmatrix} \vdots \\ 20.00 \\ 20.00 \\ 20.00 \\ 20.00 \\ 20.00 \\ 20.00 \\ 20.00 \\ 20.00 \\ 22.00 \\ 22.00 \\ 22.00 \\ \vdots \end{bmatrix} mm$$

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This function calculates the force using all 125 combinations of Speed, HorsePower, and Diameter and returns a column vector.

$$\text{Torque\_Force}(\text{speed}, \text{horse}, \text{diam}, \text{indices}) := \begin{array}{l} \text{for } i \in ORIGIN..\text{rows}(\text{indices}) \\ \quad \text{s\_idx} \leftarrow \text{indices}_{i, ORIGIN} \\ \quad \text{h\_idx} \leftarrow \text{indices}_{i, ORIGIN+1} \\ \quad \text{d\_idx} \leftarrow \text{indices}_{i, ORIGIN+2} \\ \quad R \leftarrow \text{speed}_{\text{s\_idx}} \\ \quad h \leftarrow \text{horse}_{\text{h\_idx}} \\ \quad d \leftarrow \text{diam}_{\text{d\_idx}} \\ \quad r \leftarrow \dfrac{d}{2} \\ \quad T_i \leftarrow \dfrac{h}{R \cdot 2 \cdot \pi} \\ \quad \text{sforce}_i \leftarrow \dfrac{T_i}{r} \\ \quad X_{i, ORIGIN} \leftarrow T_i \\ \quad X_{i, ORIGIN+1} \leftarrow \text{sforce}_i \\ X \end{array}$$

$$\text{Data} := \text{Torque\_Force}(\text{Speed}, \text{HorsePower}, \text{Diameter}, \text{Indices})$$

$$\text{Torque} := \text{Data}^{\langle ORIGIN \rangle} = \begin{bmatrix} 26.11 \\ 24.10 \\ 22.38 \\ 20.89 \\ 19.58 \\ 30.86 \\ 28.48 \\ 26.45 \\ 24.69 \\ 23.14 \\ 35.60 \\ 32.87 \\ \vdots \end{bmatrix} N \cdot m \qquad \text{Force} := \text{Data}^{\langle ORIGIN+1 \rangle} = \begin{bmatrix} 2611.00 \\ 2410.15 \\ 2238.00 \\ 2088.80 \\ 1958.25 \\ 3085.73 \\ 2848.36 \\ 2644.91 \\ 2468.58 \\ 2314.30 \\ 3560.45 \\ 3286.57 \\ \vdots \end{bmatrix} N$$

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What is the maximum force and torque created?

$max(\mathrm{Force}) = 4509.91\ N$ $\quad max(\mathrm{Torque}) = 45.10\ N \cdot m$

Now, let's find the combination of Speed, HorsePower, and Diameter that created this maximum force.

$\mathrm{imax} := max(\mathrm{Force}) = 4509.91\ N$

Use the ***match*** function to find the index of the maximum force.

$\mathrm{idx} := match(\mathrm{imax}, \mathrm{Force}) = [21.00]$

idx is 1x1 matrix. Use ORIGIN to create a scalar. $\quad \mathrm{idx} := \mathrm{idx}_{ORIGIN} = 21.00$

$\mathrm{max\_force\_value} := \mathrm{Force}_{\mathrm{idx}} = 4509.91\ N$

Find the torque ath this value of force

$\mathrm{max\_torque\_value} := \mathrm{Torque}_{\mathrm{idx}} = 45.10\ N \cdot m$

We calculated the index number (idx) for the maximum force. Use this index to find the corresponding values of Speed, HorsePower, and Diameter.

The maximum force occurs when the following occurs:

$\mathrm{Speed\_Val} := \mathrm{Input\_Data}_{\mathrm{idx}, ORIGIN} = 300.00\ rpm$

$\mathrm{HorsePower\_Val} := \mathrm{Input\_Data}_{\mathrm{idx}, ORIGIN+1} = 1.90\ hp$

$\mathrm{Diameter\_Val} := \mathrm{Input\_Data}_{\mathrm{idx}, ORIGIN+2} = 20.00\ mm$

Add the matrix Data to the Input_Data matrix by using the ***augment*** function.

$\text{Total\_Data} := \underline{augment}(\text{Input\_Data}, \text{Data})$

Verify that the above results are accurate by looking at Total_Data in decending order.

$\text{Sorted\_Data} := \underline{reverse}(\underline{csort}(\text{Total\_Data}, ORIGIN + 4))$

$$\text{Sorted\_Data} = \begin{array}{r} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ \vdots \\ 124 \end{array} \begin{bmatrix} 5 \, \frac{1}{s} & 1416.83 \, W & 0.02 \, m & 45.099 \, J & 4509.909 \, N \\ 5.417 \, \frac{1}{s} & 1416.83 \, W & 0.02 \, m & 41.63 \, J & 4162.993 \, N \\ 5 \, \frac{1}{s} & 1416.83 \, W & 0.022 \, m & 45.099 \, J & 4099.917 \, N \\ 5 \, \frac{1}{s} & 1267.69 \, W & 0.02 \, m & 40.352 \, J & 4035.182 \, N \\ 5.833 \, \frac{1}{s} & 1416.83 \, W & 0.02 \, m & 38.656 \, J & 3865.636 \, N \\ & & & \vdots & \end{bmatrix}$$

$$\text{Sorted\_Data}^{\langle 1 \rangle} = [300.00 \;\; 1.90 \;\; 20.00 \;\; 45.10 \;\; 4509.91] \, [rpm \;\; hp \;\; mm \;\; N \cdot m \;\; N]$$

$$\text{Sorted\_Data} = \begin{array}{r} 0 \\ \vdots \\ 120 \\ 121 \\ 122 \\ 123 \\ 124 \end{array} \begin{bmatrix} \vdots & & & & \\ 6.250 \, \frac{1}{s} & 820.270 \, W & 0.026 \, m & 20.888 \, J & 1606.769 \, N \\ 5.833 \, \frac{1}{s} & 820.270 \, W & 0.028 \, m & 22.380 \, J & 1598.571 \, N \\ 6.667 \, \frac{1}{s} & 820.270 \, W & 0.026 \, m & 19.583 \, J & 1506.346 \, N \\ 6.250 \, \frac{1}{s} & 820.270 \, W & 0.028 \, m & 20.888 \, J & 1492.000 \, N \\ 6.667 \, \frac{1}{s} & 820.270 \, W & 0.028 \, m & 19.583 \, J & 1398.750 \, N \end{bmatrix}$$

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Test the sorted data. Use the ***vectorize*** operator to test each result.

$$TorqueTest := \overrightarrow{Test1\left(Sorted\_Data^{\langle ORIGIN+3\rangle}\right)} = \begin{array}{c|c} 0 & \text{“Meets”} \\ 1 & \text{“Meets”} \\ 2 & \text{“Meets”} \\ 3 & \text{“Meets”} \\ 4 & \text{“Meets”} \\ 5 & \text{“Meets”} \\ \vdots & \vdots \end{array}$$

$$ForceTest := \overrightarrow{Test2\left(Sorted\_Data^{\langle ORIGIN+4\rangle}\right)} = \begin{array}{c|c} 0 & \text{“Meets”} \\ 1 & \text{“Meets”} \\ 2 & \text{“Meets”} \\ 3 & \text{“Meets”} \\ 4 & \text{“Meets”} \\ 5 & \text{“Meets”} \\ \vdots & \vdots \end{array}$$

Use the augment function to add the TorqueTest and ForceTest vectors to the Sorted_Data matrix.

$$Sorted\_Data := augment(Sorted\_Data, TorqueTest, ForceTest)$$

$$Sorted\_Data = \begin{array}{c|cccccccc} 0 & 5.00\,\frac{1}{s} & 1416.83\,W & 0.02\,m & 45.10\,J & 4509.91\,N & \text{“Meets”} & \text{“Meets”} \\ 1 & 5.42\,\frac{1}{s} & 1416.83\,W & 0.02\,m & 41.63\,J & 4162.99\,N & \text{“Meets”} & \text{“Meets”} \\ 2 & 5.00\,\frac{1}{s} & 1416.83\,W & 0.02\,m & 45.10\,J & 4099.92\,N & \text{“Meets”} & \text{“Meets”} \\ 3 & 5.00\,\frac{1}{s} & 1267.69\,W & 0.02\,m & 40.35\,J & 4035.18\,N & \text{“Meets”} & \text{“Meets”} \\ \vdots & & & & & & & \vdots \end{array}$$

$$Sorted\_Data = \begin{array}{c|cccccccc} \vdots & \vdots \\ 75 & 5.00\,\frac{1}{s} & 969.41\,W & 0.03\,m & 30.86\,J & 2373.64\,N & \text{“Meets”} & \text{“Meets”} \\ 76 & 6.25\,\frac{1}{s} & 1118.55\,W & 0.02\,m & 28.48\,J & 2373.64\,N & \text{“Doesn't Meet”} & \text{“Meets”} \\ 77 & 5.42\,\frac{1}{s} & 1118.55\,W & 0.03\,m & 32.87\,J & 2347.55\,N & \text{“Meets”} & \text{“Meets”} \\ 78 & 5.83\,\frac{1}{s} & 1118.55\,W & 0.03\,m & 30.52\,J & 2347.55\,N & \text{“Meets”} & \text{“Meets”} \\ 79 & 6.67\,\frac{1}{s} & 1267.69\,W & 0.03\,m & 30.26\,J & 2327.99\,N & \text{“Meets”} & \text{“Meets”} \\ \vdots & & & & & & & \vdots \end{array}$$

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Display the Sorted_Data matrix in an Excel component.
Double click to view the data in Excel.

Inputs

$$\mathbf{excel}_{\text{“A1”}} := \begin{bmatrix} \text{“Speed”} & \text{“Horse Power”} & \text{“Diameter”} & \text{“Torque”} & \text{“Force”} & \text{“TorqueTest”} & \text{“ForceTest”} \\ \text{“rpm”} & \text{“hp”} & \text{“mm”} & \text{“N*m”} & \text{“N”} & NaN & NaN \end{bmatrix}$$

$$\mathbf{excel}_{\text{“A3”}} := \frac{Sorted\_Data^{\langle ORIGIN \rangle}}{rpm} \quad \mathbf{excel}_{\text{“B3”}} := \frac{Sorted\_Data^{\langle ORIGIN+1 \rangle}}{hp} \quad \mathbf{excel}_{\text{“C3”}} := \frac{Sorted\_Data^{\langle ORIGIN+2 \rangle}}{mm}$$

$$\mathbf{excel}_{\text{“D3”}} := \frac{Sorted\_Data^{\langle ORIGIN+3 \rangle}}{N \cdot m} \quad \mathbf{excel}_{\text{“E3”}} := \frac{Sorted\_Data^{\langle ORIGIN+4 \rangle}}{N}$$

$$\mathbf{excel}_{\text{“F3”}} := Sorted\_Data^{\langle ORIGIN+5 \rangle} \quad \mathbf{excel}_{\text{“G3”}} := Sorted\_Data^{\langle ORIGIN+6 \rangle}$$

Speed	Horse Power	Diameter	Torque	Force	TorqueTest	ForceTest
rpm	hp	mm	N*m	N		
300	1.9	20	45.10	4509.91	Meets	Meets
325	1.9	20	41.63	4162.99	Meets	Meets
300	1.9	22	45.10	4099.92	Meets	Meets
300	1.7	20	40.35	4035.18	Meets	Meets
350	1.9	20	38.66	3865.64	Meets	Meets
325	1.9	22	41.63	3784.54	Meets	Meets
300	1.9	24	45.10	3758.26	Meets	Meets
325	1.7	20	37.25	3724.78	Meets	Meets
300	1.7	22	40.35	3668.35	Meets	Meets
375	1.9	20	36.08	3607.93	Meets	Meets
300	1.5	20	35.60	3560.45	Meets	Meets
350	1.9	22	38.66	3514.21	Meets	Meets
300	1.9	26	45.10	3469.16	Meets	Meets
325	1.9	24	41.63	3469.16	Meets	Meets
350	1.7	20	34.59	3458.73	Meets	Meets
325	1.7	22	37.25	3386.17	Meets	Meets
400	1.9	20	33.82	3382.43	Meets	Meets
300	1.7	24	40.35	3362.65	Meets	Meets
325	1.5	20	32.87	3286.57	Meets	Meets
375	1.9	22	36.08	3279.93	Meets	Meets
300	1.5	22	35.60	3236.78	Meets	Meets

Out...