Engineering Example A3:
Propagation Delay on a Printed Circuit Board

This worksheet calculates the propagation delay time for a signal traveling along a microstrip copper trace on a printed circuit board.

Field: Electrical Engineering

Features used:
User-defined function
Range variable
Plotting
Solve block
Excel Component

This example is based on an example provided by John Sheehan of PTC.

Engineering Example A3
Propagation Delay on a Printed Circuit Board

PROBLEM

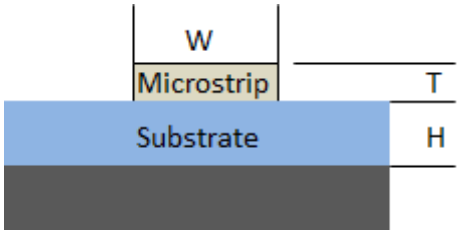
Calculate the propagation delay time for a signal traveling along a microstrip copper trace on a printed circuit board. Use the following formulas and values:

$$Z_o = \frac{87}{\sqrt{\varepsilon_r + 1.41}} \cdot ln\left(\frac{5.98 \cdot H}{0.8 \cdot W + T}\right)$$ Characteristic Impedance Ω

$$C_o = \frac{0.67 \cdot (\varepsilon_r + 1.41)}{ln\left(\frac{5.98 \cdot H}{0.8 \cdot W + T}\right)}$$ Intrinsic Capacitance pF/in

$$t_{pd} = 1.017 \cdot \sqrt{0.475 \cdot \varepsilon_r + 0.67}$$ Uncorrected Propagation Delay (ns/in)

$$C_f = \sqrt{1 + \frac{C_d}{C_o \cdot Length}}$$ Correction Factor (to account for the capacitance of device loads)



Given:

$\varepsilon_r := 4.2$ Dielectric Coefficient of Substrate

$H := 10\ mil$ Height of substrate

$C_d := 10 \cdot pF$ Capacitance due to device loads

Define Variables:

Trace Length: $Length := 7\ ft$

Trace Width: $W := 28\ mm$

Trace Thickness: $T := 5\ mil$

Engineering Example A3
Propagation Delay on a Printed Circuit Board

Microstrip Calculations:

Characteristic Impedance of the microstrip trace:

$$Z_o := \frac{87}{\sqrt{\varepsilon_r + 1.41}} \cdot ln\left(\frac{5.98 \cdot H}{0.8 \cdot W + T}\right) \cdot ohm = -99.05\ \Omega$$

Use the "explicit" keyword to substitute and show the variable values.

$$\frac{87}{\sqrt{\varepsilon_r + 1.41}} \cdot ln\left(\frac{5.98 \cdot H}{0.8 \cdot W + T}\right) \cdot ohm \xrightarrow{explicit, \varepsilon_r, H, W, T} \frac{87}{\sqrt{4.2 + 1.41}} \cdot ln\left(\frac{5.98 \cdot 10\ mil}{0.8 \cdot 28\ mm + 5\ mil}\right) \cdot ohm$$

Expressions
(Use "Variable" Label on all variables

Intrinsic capacitance per unit length of the trace:

$$C_o := \frac{0.67 \cdot (\varepsilon_r + 1.41)}{ln\left(\frac{5.98 \cdot H}{0.8 \cdot W + T}\right)} \cdot \frac{pF}{in} = -1.39\ \frac{pF}{in}$$

Propagation Delay

$$t_{pd} := 1.017 \cdot \sqrt{0.475 \cdot \varepsilon_r + 0.67} \cdot \frac{ns}{in} = 1.6602\ \frac{ns}{in}$$

Correction Factor

$$C_f := \sqrt{1 + \frac{C_d}{C_o \cdot Length}} = 0.9563$$

Corrected Propagation Delay

$$t'_{pd} := t_{pd} \cdot C_f = 1.5878\ \frac{ns}{in}$$

Functions
(Use "Function" Label on function name. and "Variable" Label on others. These functions will be used later.)

$$C_o(\varepsilon_r, H, W, T) := \frac{0.67 \cdot (\varepsilon_r + 1.41)}{ln\left(\frac{5.98 \cdot H}{0.8 \cdot W + T}\right)} \cdot \frac{pF}{in}$$

$$C_o(\varepsilon_r, H, W, T) = -1.39\ \frac{pF}{in}$$

$$t_{pd}(\varepsilon_r) := 1.017 \cdot \sqrt{0.475 \cdot \varepsilon_r + 0.67} \cdot \frac{ns}{in}$$

$$t_{pd}(\varepsilon_r) = 1.66\ \frac{ns}{in}$$

$$C_f(C_d, C_o, L) := \sqrt{1 + \frac{C_d}{C_o \cdot L}}$$

$$C_f(C_d, C_o(\varepsilon_r, H, W, T), Length) = 0.9563$$

$$t'_{pd}(t_{pd}, C_f) := t_{pd} \cdot C_f$$

$$t'_{pd}(t_{pd}(\varepsilon_r), C_f(C_d, C_o, Length)) = 1.5878\ \frac{ns}{in}$$

Total Propagation Delay for Entire 7 ft Length of Copper:

$$T_{PD\_Total} := t'_{pd} \cdot Length = 133.371069\ ns$$

[1.00]

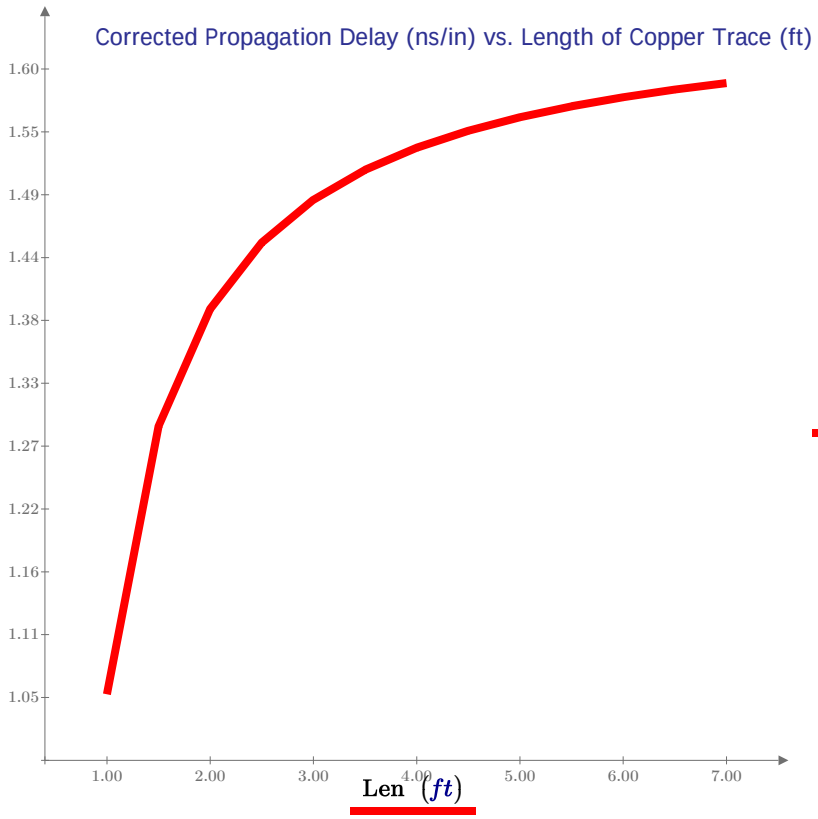
Engineering Example A3
Propagation Delay on a Printed Circuit Board

Calculate Propogation Delay as a function of time

Create a vector for Length $\text{Len} := 12\ in, 18\ in .. \text{Length} = \begin{bmatrix} 1.00 \\ 1.50 \\ 2.00 \\ 2.50 \\ 3.00 \\ 3.50 \\ 4.00 \\ 4.50 \\ 5.00 \\ 5.50 \\ 6.00 \\ 6.50 \\ 7.00 \end{bmatrix} ft$

$$\text{CorrectedPropDelay} := t_{pd} \cdot \underline{C_f}(C_d, C_o, \text{Len}) = \begin{bmatrix} 1.0528 \\ 1.2875 \\ 1.3901 \\ 1.4482 \\ 1.4856 \\ 1.5118 \\ 1.5311 \\ 1.5460 \\ 1.5578 \\ 1.5674 \\ 1.5753 \\ 1.5820 \\ 1.5878 \end{bmatrix} \frac{ns}{in}$$

Corrected Propagation Delay (ns/in) vs. Length of Copper Trace (ft)



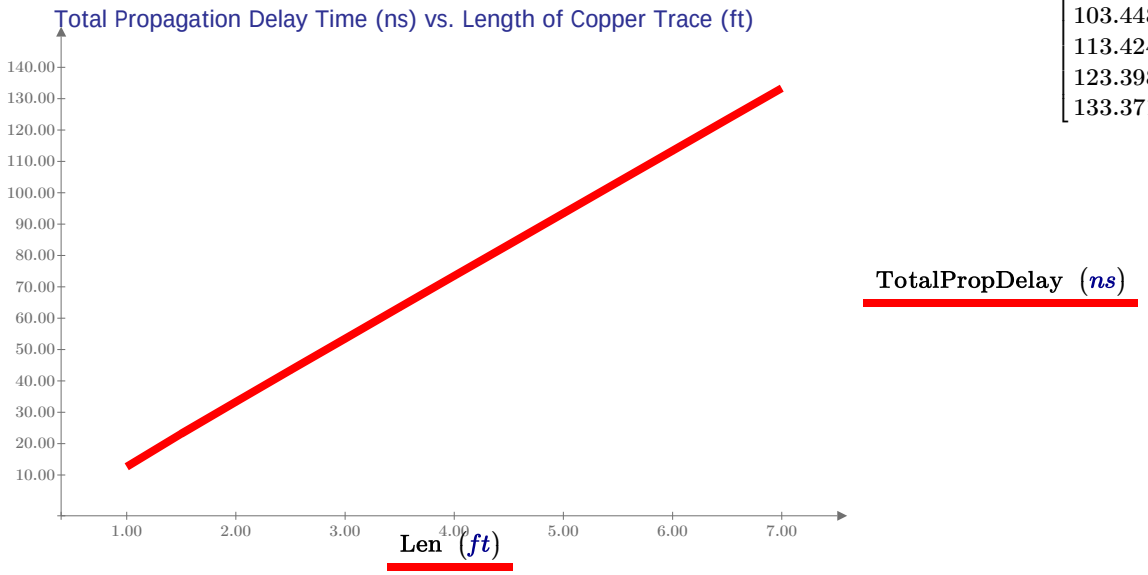
CorrectedPropDelay $\left(\frac{ns}{in}\right)$

$\begin{bmatrix} 12.6336 \end{bmatrix}$

Engineering Example A3
Propagation Delay on a Printed Circuit Board

Plot Total Propagation Delay Time as a function of length

$$\text{TotalPropDelay} := \overrightarrow{t_{pd} \cdot \mathcal{C}_f(C_d, C_o, \text{Len}) \cdot \text{Len}} = \begin{bmatrix} 12.6336 \\ 23.1754 \\ 33.3625 \\ 43.4450 \\ 53.4821 \\ 63.4954 \\ 73.4946 \\ 83.4848 \\ 93.4688 \\ 103.4485 \\ 113.4249 \\ 123.3989 \\ 133.3711 \end{bmatrix} ns$$



$$\text{excel}_{\text{"A2"}} := \frac{CorrectedPropDelay}{\frac{ns}{in}} \qquad \text{excel}_{\text{"B2"}} := \frac{Len}{in}$$

Corrected	Length	Product	Difference
1.0528	12	12.6336	
1.2875	18	23.1754	10.5418
1.3901	24	33.3625	10.1870
1.4482	30	43.4450	10.0825
1.4856	36	53.4821	10.0371
1.5118	42	63.4954	10.0133
1.5311	48	73.4946	9.9992
1.5460	54	83.4848	9.9902
1.5578	60	93.4688	9.9840
1.5674	66	103.4485	9.9797
1.5753	72	113.4249	9.9765
1.5820	78	123.3989	9.9740
1.5878	84	133.3711	9.9721

The Corrected Propagation Delay is a curved line, yet the Total Propagation Delay looks like a straight line.

Use the Excel Component to check the difference between each value.

Engineering Example A3
Propagation Delay on a Printed Circuit Board

Now, calculate the width of the trace to reduce the total propagation delay time to 132 ns.

Because W is an unknown, it must be included in a function.

Guess Values

$t_{pd} := 1.66 \frac{ns}{in}$ $\quad$ $Length := 7.00\ ft$

$H := 10.00\ mil$ $\quad$ $T := 0.13\ mm$

Constraints

$132\ ns = t_{pd} \cdot \underline{C_f}\left(C_d, \underline{C_o}\left(\varepsilon_r, H, W, T\right), Length\right) \cdot Length$

Solver

$Width := \text{find}(W) = 50.67\ mm$

Check

$NewC_o := \underline{C_o}\left(\varepsilon_r, H, Width, T\right) = -1.14 \frac{pF}{in}$

$Check := t_{pd} \cdot \underline{C_f}\left(C_d, NewC_o, Length\right) \cdot Length = 132.00\ ns$ OK

$Width = 50.67\ mm$

The width of trace must increase from $W = 28.00\ mm$ to $Width = 50.67\ mm$ in order to reduce the total propagation delay time from $T_{PD\_Total} = 133.37\ ns$ to 132 ns.